KS5 "Full Coverage": Rational Expressions (Manipulating Expressions, Indices and Surds)

This worksheet is designed to cover one question of each type seen in past papers, for each GCSE Higher Tier topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise → Past Papers* (or *Library → Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Combine multiple methods of simple factorisation.

[Edexcel C1 May 2015 Q8a]

Factorise completely $9x - 4x^3$.

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Question 2

Categorisation: As above, but including factorisation of a quadratic.

[Edexcel C1 Jan 2006 Q1]

Factorise completely $x^3 - 4x^2 + 3x$

.....

Question 3

Categorisation: Expand expressions involving roots.

[Edexcel C1 Jan 2007 Q6a]

Show that $\left(4 + 3\sqrt{x}\right)^2$ can be written as $16 + k\sqrt{x} + 9x$. where k is a constant to be found.

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Question 4

Categorisation: Split fractions and write roots as powers to simplify.

[Edexcel C1 Jan 2009 Q6a]

Given that $\frac{2x^2-x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$, write down the value of p and the value of q.

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Question 5

Categorisation: As above, but with expanding.

[Edexcel C1 May 2005 Q7a Edited]

Show that $\frac{(3-\sqrt{x})^2}{\sqrt{x}}$ can be written as $ax^{-\frac{1}{2}} - b + x^{\frac{1}{2}}$, where a and b are integers to be found.

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Question 6

Categorisation: Expand and refactorise expressions.

[Edexcel C1 May 2006 Q9a]

Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$, express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants.

$f(x) =$ .....

Question 7

Categorisation: Raise a number to a fractional exponent.

[Edexcel C1 Jan 2009 Q1a]

Write down the value of $125^{\frac{1}{3}}$

.....

Question 8

Categorisation: As above but negative.

[Edexcel C1 Jan 2009 Q1b]

Find the value of $125^{-\frac{2}{3}}$

.....

Question 9

Categorisation: Change the base of a power.

[Edexcel C1 May 2016 Q2]

Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

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Question 10

Categorisation: As above.

[Edexcel C1 May 2015 Q7a]

Given that $y = 2^x$, express 4^x in terms of y.

$4^x =$ .....

Question 11

Categorisation: Solve a quadratic-like equation by first changing the base (no logs required).

[Edexcel C1 May 2015 Q7b]

Solve $8(4^x) - 9(2^x) + 1 = 0$.

.....

Question 12

Categorisation: Simplify an expression involving different bases.

[Edexcel C1 May 2013(R) Q5b]

Solve $2^x \times 4^{x+1} = 8$

.....

Question 13

Categorisation: Write a term as a single power.

[Edexcel C1 June 2009 Q2]

Given that $32\sqrt{2} = 2^a$, find the value of a .

.....

Question 14

Categorisation: Raise an algebraic term to a power.

[Edexcel C1 Jan 2008 Q2b]

Simplify $(16x^{12})^{\frac{3}{4}}$

.....

Question 15

Categorisation: As above.

[Edexcel C1 Jan 2011 Q1b]

Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$

.....

Question 16

Categorisation: As above, but involving a fraction.

[Edexcel C1 May 2012 Q2b]

Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

.....

Question 17

Categorisation: Raise a surd to a power.

[Edexcel C1 May 2015 Q1a]

Simplify $\left(2\sqrt{5}\right)^2$

.....

Question 18

Categorisation: Add or subtract surds by first simplifying.

[Edexcel C1 May 2016 Q3a]

Simplify $\sqrt{50} - \sqrt{18}$, giving your answer in the form $a\sqrt{2}$, where a is an integer.

.....

Question 19

Categorisation: Expand brackets involving surds.

[Edexcel C1 Jan 2013 Q3i]

Express

$$(5 - \sqrt{8})(1 + \sqrt{2})$$

in the form $a + b\sqrt{2}$, where a and b are integers.

.....

Question 20

Categorisation: Rationalising a simple denominator.

[Edexcel C1 May 2013(R) Q2]

Find $\frac{15}{\sqrt{3}} - \sqrt{27}$ in the form $k\sqrt{3}$, where k is an integer.

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Question 21

Categorisation: Rationalise the denominator where the denominator involves multiple terms.

[Edexcel C1 May 2013 Q1] Simplify

$$\frac{7+\sqrt{5}}{\sqrt{5}-1}$$

giving your answer in the form $a+b\sqrt{5}$, where a and b are integers.

.....

Question 22

Categorisation: As above, but a more complicated denominator.

[Edexcel C1 May 2015 Q1b]

Simplify $\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$, giving your answer in the form $a+\sqrt{b}$, where a and b are integers.

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Question 23

Categorisation: As above but where prior simplification of the denominator is needed.

[Edexcel C1 May 2016 Q3b] By first simplifying $\sqrt{50} - \sqrt{18}$ or otherwise, simplify:

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b\ ne\ 1$.

.....

Question 24

Categorisation: Solve a linear equation involving surds.

[Edexcel C1 May 2014(R) Q5] Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

$x =$ .....

Question 25

Categorisation: Divide surds in context.

[Edexcel C1 May 2014 Q6b]

A rectangle R has a length $\left(1 + \sqrt{5}\right)$ cm and an area of $\sqrt{80}$ cm ^{2}.

Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

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Answers

Question 1

$x(3+2x)(3-2x)$

Question 2

$x(x-3)(x-1)$

Question 3

$k = 24$

Question 4

$p = \frac{3}{2}, q = 1$

Question 5

$9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$

Question 6

$f(x) = x(x^2 - 8x + 15)$

Question 7

5

Question 8

$\frac{1}{25}$

Question 9

$a = 6\,, b = 2$

Question 10

$4^x = y^2$

Question 11

$x = -3$ or $x = 0$

Question 12

$x = \frac{1}{3}$

Question 13

$a = \frac{11}{2}$

Question 14

$8x^9$

Question 15

16

Question 16

$\frac{2}{5x^2}$

Question 17

20

Question 18

$2\sqrt{2}$

Question 19

$1 + 3\sqrt{2}$

Question 20

$2\sqrt{3}$

Question 21

$3 + 2\sqrt{5}$

Question 22

$3 + \sqrt{10}$

Question 23

$3\sqrt{6}$

Question 24

$x = 5\sqrt{2}$

Question 25

$5 - \sqrt{5}$