Advanced Mathematics Support Programme®

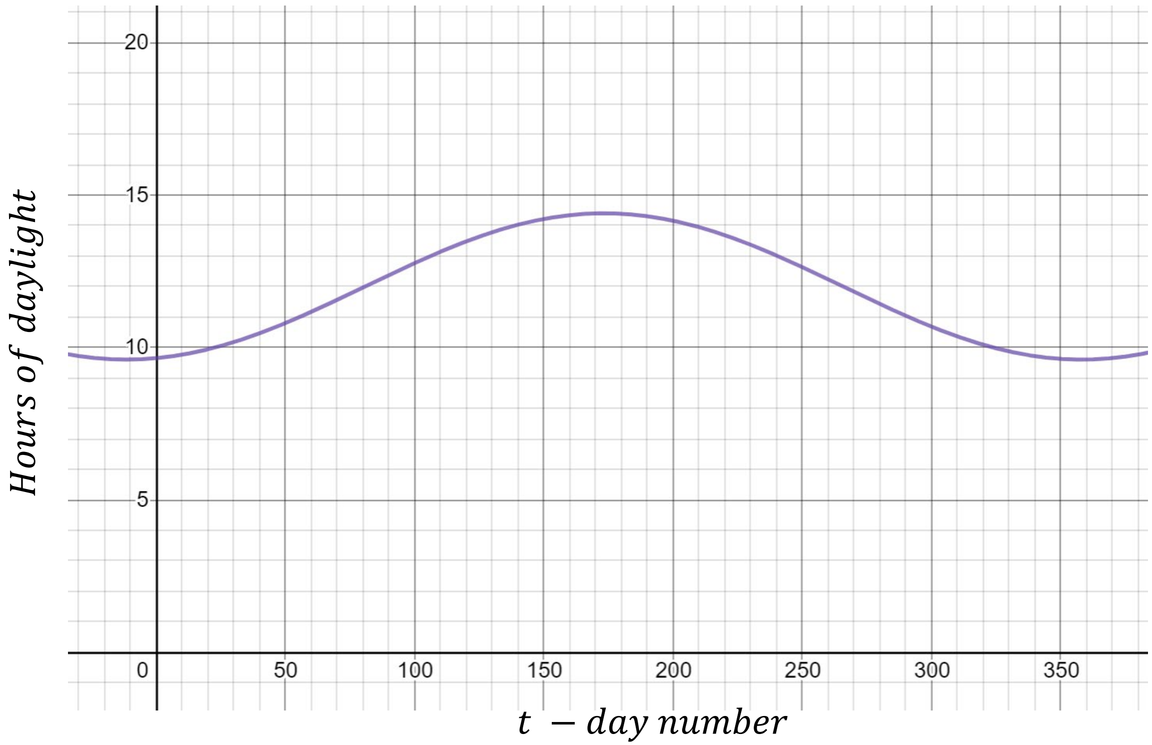
Did you know?

Sunrise and sunset times are modelled using trigonometrical equations

For San Diego, California, a simple equation to model daylight hours would be:

$$\text{Number of daylight hours} = 2.4\sin(0.0017t - 1.377) + 12$$

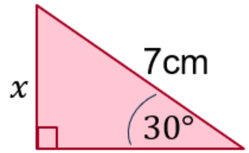
where t is the day of year from 0 to 365



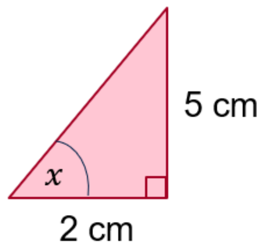
From the graph can you tell which dates of the year are the shortest and longest day?

Solving equations with Trigonometry

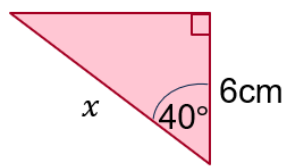
1. Calculate the length of the side marked x in this triangle.



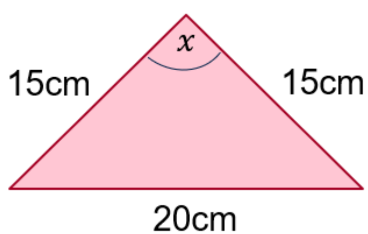
2. Calculate the value of the angle marked x in this triangle.



3. Calculate the value of the side marked x in this triangle

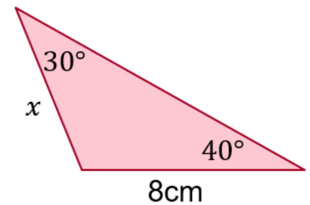


4. Calculate the value of the angle marked x in this triangle.



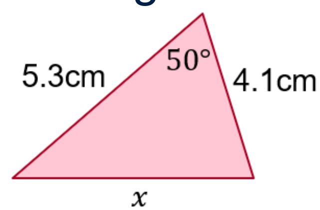
5. Calculate the value of the side marked x in this triangle

Sine rule

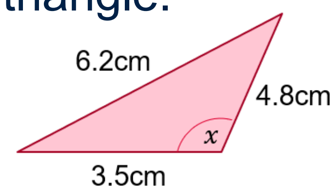


6. Calculate the value of the side marked x in this triangle.

Cosine rule

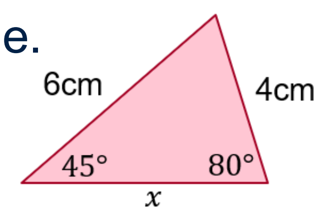


7. Calculate the value of the angle marked x in this triangle.



8. Calculate the value of the side marked x in this triangle.

Sine rule

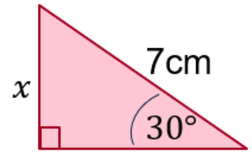


Solving equations with Trigonometry

Solutions on the next slide….

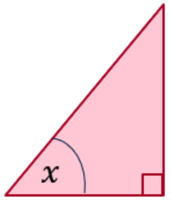
Solving equations with Trigonometry Solutions

1. Calculate the length of the side marked x in this triangle.



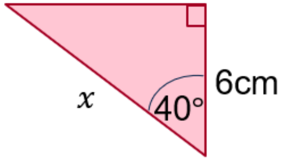
$$\sin 30 = \frac{x}{7}$$
$$x = 7 \times \sin 30$$
$$x = 3.5cm$$

2. Calculate the value of the angle marked x in this triangle.



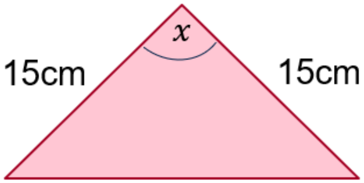
$$\tan x = \frac{5}{2}$$
$$x = \tan^{-1}\left(\frac{5}{2}\right)$$
$$x = 68.2° \text{ to } 1 d.p$$

3. Calculate the value of the side marked x in this triangle

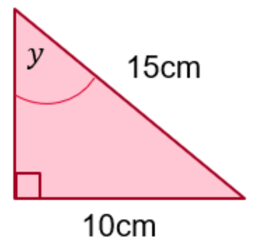


$$\cos 40 = \frac{6}{x}$$
$$x\cos 40 = 6$$
$$x = \frac{6}{\cos 40}$$
$$x = 7.8 \text{ to } 1 \; d.p$$

4. Calculate the value of the angle marked x in this triangle.

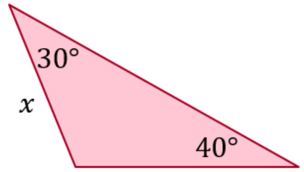


$$\sin y = \frac{10}{15}$$
$$y = \sin^{-1}\left(\frac{10}{15}\right)$$
$$y = 41.81°$$
$$x = 2y$$
$$x = 83.6° \text{ to } 1 \; d.p$$



Solving equations with Trigonometry Solutions

5. Calculate the value of the side marked x in this triangle

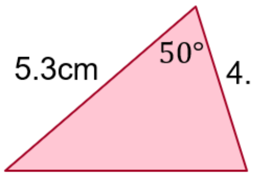


$$\frac{x}{sin40} = \frac{8}{sin30}$$

$$x = \frac{8 \times sin40}{sin30}$$

$$x = 10.3\ cm\ to\ 1\ d.p$$

6. Calculate the value of the side marked x in this triangle.

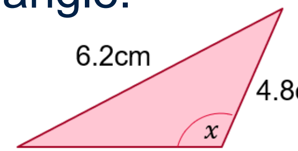


$$x^2 = 4.1^2 + 5.3^2 - (2 \times 4.1 \times 5.3 \times cos50)$$

$$x^2 = 16.96 \ldots$$

$$x = 4.1\ to\ 1\ d.p$$

7. Calculate the value of the angle marked x in this triangle.



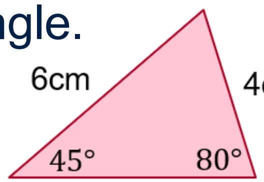
Rearrange the cosine rule formula first

$$cos\ x = \frac{3.5^2 + 4.8^2 - 6.2^2}{2 \times 3.5 \times 4.8}$$

$$cos\ x = \frac{-3}{32}$$

$$x = cos^{-1}\left(\frac{-3}{32}\right) \qquad x = 95.4°\ to\ 1\ d.p$$

8. Calculate the value of the side marked x in this triangle.



$$\frac{x}{sin55} = \frac{6}{sin80}$$ (The third angle is 55°)

$$x = \frac{6 \times sin55}{sin80} \qquad x = 5.0\ cm\ to\ 1\ d.p$$

Other Equations

Solve the following:

1. $3^x = 243$

2. $2^{2x+3} = 128$

3. $\sqrt{x+3} = 7$

4. $2\sqrt{x} + 1 = \sqrt{12} + 3$

5. $3\sqrt{x} + 12 = 7\sqrt{x}$

6. $\sin x = \frac{1}{2} \qquad 0 \leq x \leq 360$

7. $\cos x = 0.866 \qquad 0 \leq x \leq 360$

8. $\frac{8}{3x+7} = 2$

Other Equations

Solutions on the next slide….

Other Equations Solutions

1. $3^x = 243$

$3^5 = 243$
$3^x = 3^5$
$x = 5$

2. $2^{2x+3} = 128$

$2^{2x+3} = 2^7$
$2x + 3 = 7$
$2x = 4$
$x = 2$

3. $\sqrt{x+3} = 7$

Squaring gives
$x + 3 = 49$
$x = 46$

4. $2\sqrt{x} + 1 = \sqrt{12} + 3$

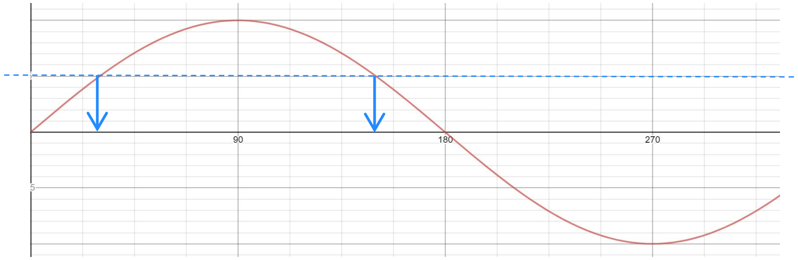
$2\sqrt{x} = \sqrt{12} + 2$
$2\sqrt{x} = 2\sqrt{3} + 2$
$\sqrt{x} = \sqrt{3} + 1$
$x = (\sqrt{3} + 1)^2$
$x = 3 + 1 + 2\sqrt{3}$
$x = 4 + 2\sqrt{3}$

Other Equations 2 Solutions

5. $3\sqrt{x}+12=7\sqrt{x}$

$12=7\sqrt{x}-3\sqrt{x}$
$12=4\sqrt{x}$
$3=\sqrt{x}$
$x=9$

6. $\sin x=\frac{1}{2}\quad 0\leq x\leq 360°$

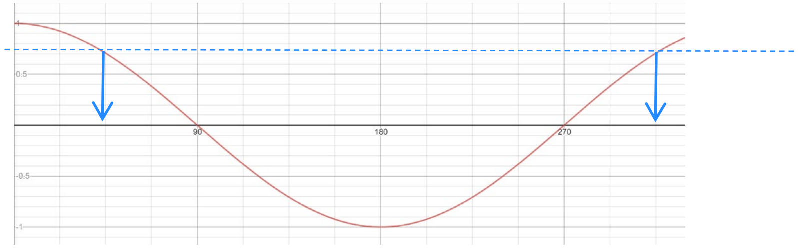


$x=sin^{-1}\left(\frac{1}{2}\right)=30°$

Using the graph and the symmetry we can see there is another value which is $180°-30°=150°$

So $x=30°\ or\ x=150°$

7. $\cos x=0.866\quad 0\leq x\leq 360°$



$x=cos^{-1}(0.866)=30°$

similarly using the graph and symmetry

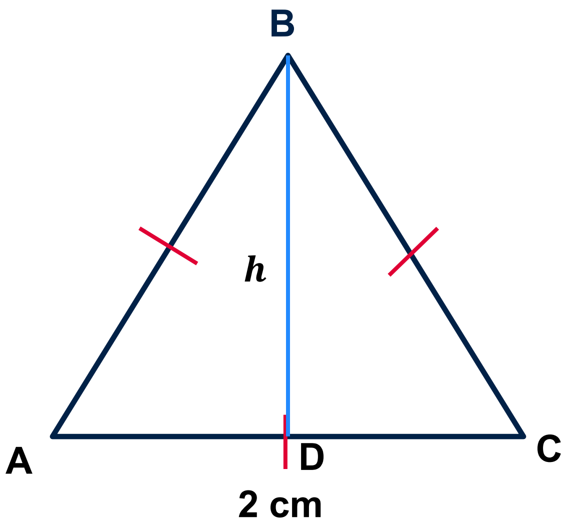
$x=360-30=330°$

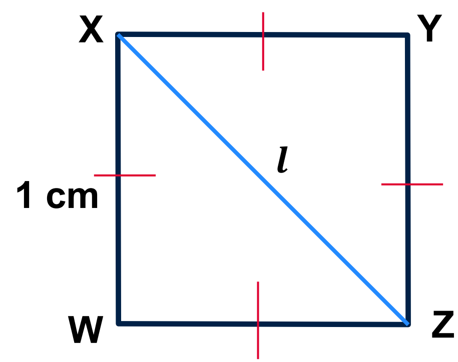
So $x=30° or\ x=330°$

8. $\frac{8}{3x+7}=2$

$8=2(3x+7)$
$8=6x+14$
$-6=6x$
$x=-1$

Missing info





	Answer
Length of AB	
Length of BD	
Length of AD	
Size of $\angle BAD$	
Size of $\angle ABD$	

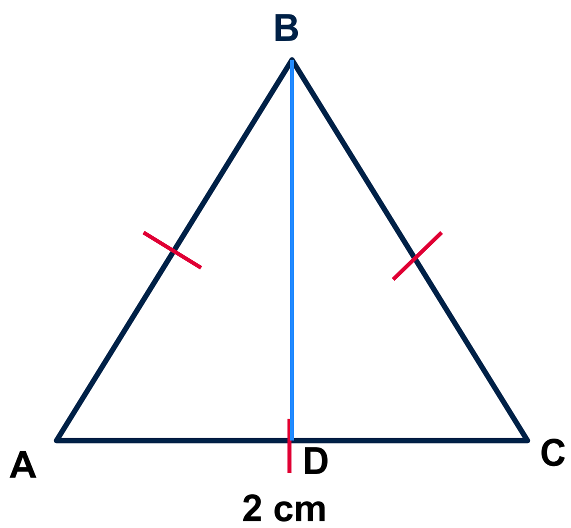
	Answer
Length of WZ	
Length of XZ	
Size of $\angle WZX$	
Size of $\angle WXZ$	

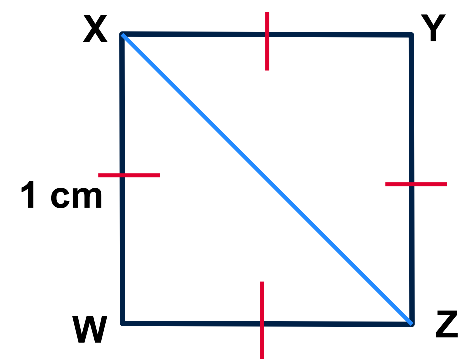
Use your knowledge of regular shapes to complete the tables above (you will need them for the next task).

Missing info

Solutions on the next slide….

Missing info Solution





	Answer
Length of AB	$2\ cm$
Length of BD	$\sqrt{3}$ *
Length of AD	$1\ cm$
Size of $\angle BAD$	60^0
Size of $\angle ABD$	30^0

	Answer
Length of WZ	$1\ cm$
Length of XZ	$\sqrt{2}\ cm$
Size of $\angle WZX$	45^0
Size of $\angle WXZ$	45^0

* By Pythagoras' theorem $\text{BD}^2 = AB^2 - AD^2$

So $BD = \sqrt{2^2 - 1^2} = \sqrt{3}$

Let's get Triggy

Use your tables and diagrams from the previous activity to complete this table

θ	$30°$	$45°$	$60°$
$sin\theta$	$\frac{}{AB} = \frac{1}{2}$	$\frac{XW}{} = \frac{WZ}{XZ} = \frac{}{}$	$\frac{}{AB} = \frac{}{}$
$cos\theta$	$\frac{}{} = \frac{\sqrt{3}}{}$	$\frac{}{} = \frac{WZ}{} = \frac{}{}$	$\frac{}{} = \frac{}{}$
$tan\theta$	$\frac{}{} = \frac{}{\sqrt{3}}$	$\frac{}{} = \frac{}{} = 1$	$\frac{}{} = \frac{}{1} = \sqrt{}$

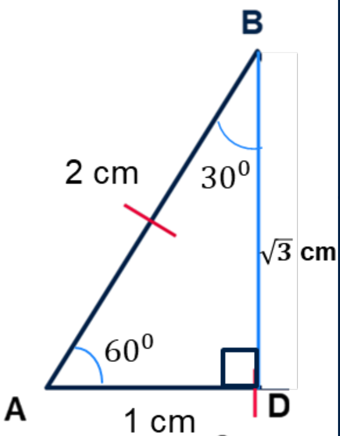
Hint available on next slide

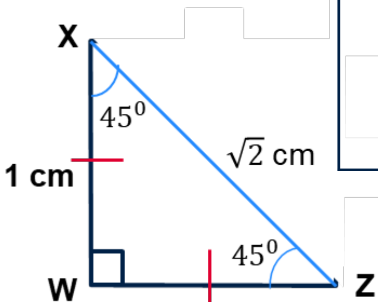
Let's get Triggy Hint

Use your tables and diagrams from the previous activity to complete this table

Some examples are filled in to get you started

These will help





θ	$30°$	$45°$	$60°$
$sin\theta =$	$\frac{AD}{AB} = \frac{1}{2}$	$\frac{XW}{} = \frac{WZ}{XZ} = \frac{}{}$	$\frac{}{AB} = \frac{\sqrt{3}}{}$
$cos\theta =$	$\frac{}{} = \frac{\sqrt{3}}{}$	$\frac{}{} = \frac{WZ}{XW} = \frac{}{}$	$\frac{AD}{AB} = \frac{}{}$
$tan\theta =$	$\frac{}{} = \frac{}{\sqrt{3}}$	$\frac{}{} = \frac{}{} = 1$	$\frac{}{} = \frac{}{1} = \sqrt{}$

Let's get Triggy

Solutions on the next slide….

Let's get Triggy Solution

Use your tables and diagrams from the previous activity to complete this table

θ	$30°$	$45°$	$60°$
$\sin\theta$	$\frac{AD}{AB} = \frac{1}{2}$	$\frac{XW}{XZ} = \frac{WZ}{XZ} = \frac{1}{\sqrt{2}}$	$\frac{BD}{AB} = \frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{BD}{AB} = \frac{\sqrt{3}}{2}$	$\frac{WX}{WZ} = \frac{WZ}{WX} = \frac{1}{\sqrt{2}}$	$\frac{AD}{AB} = \frac{1}{2}$
$\tan\theta$	$\frac{AD}{BD} = \frac{1}{\sqrt{3}}$	$\frac{WX}{WZ} = \frac{WZ}{WX} = 1$	$\frac{BD}{AD} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Trig Maze

Starting at $\sqrt{3}$ on the left hand side of the rectangle, find your way to the right hand side by landing only on expressions that are equivalent to $\sqrt{3}$

$\frac{\tan 30°}{3}$	$\frac{9}{3^{0.5}}$	$\frac{\sqrt{18}}{\sqrt{6}}$	$\frac{1.5}{0.05}$	$\frac{\sqrt{12}}{\sqrt{2}}$	$\frac{2\sqrt{6}}{\sqrt{4}}$	$\frac{\sqrt{9}}{3^0}$
$\frac{\sqrt{27}}{3}$	$\frac{3\sqrt{3}}{\sqrt{3}}$	$2\cos 60°$	$\frac{\tan 60°}{2}$	$\frac{\sin 30°}{\cos 30°}$	$3\tan 30°$	$\frac{\sqrt{6}}{\sqrt{2}}$
$\frac{6}{\sqrt{2}}$	$\frac{\cos 60°}{\sin 60°}$	$\frac{9}{3\sqrt{3}}$	$\frac{3}{\sqrt{3}}$	$2\cos 30°$	$\frac{3+\sqrt{3}}{\sqrt{3}}-1$	$3\tan 60°$
$\sqrt{3}$	$\frac{9}{\sqrt{3}}$	$2\sin 60°$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{9}}{\sqrt{3}}$	$\frac{\sqrt{6}}{2}$	$\frac{\cos 30°}{2}$
$3^{\frac{1}{2}}$	$\tan 60°$	$\frac{\sqrt{12}}{2}$	$2\sin 30°$	$\frac{\sin 60°}{\cos 60°}$	$\frac{9^{0.5}}{3^{0.5}}$	$\frac{2\sqrt{6}}{\sqrt{8}}$
$\frac{\cos 60°}{2}$	$\frac{\sqrt{12}}{4}$	$\frac{\sin 30°}{2}$	$\frac{\sqrt{9}}{3}$	$\frac{\tan 60°}{3}$	$\frac{9\times 10^1}{3\times 10^{-1}}$	$\frac{3+\sqrt{3}}{\sqrt{3}}$

Trig Maze

Solutions on the next slide….

Trig Maze Solution

Starting at $\sqrt{3}$ on the left hand side of the rectangle, find your way to the right hand side by landing only on expressions that are equivalent to $\sqrt{3}$

$\frac{\tan 30°}{3}$	$\frac{9}{3^{0.5}}$	$\frac{\sqrt{18}}{\sqrt{6}}$	$\frac{1.5}{0.05}$	$\frac{\sqrt{12}}{\sqrt{2}}$	$\frac{2\sqrt{6}}{\sqrt{4}}$	$\frac{\sqrt{9}}{3^0}$
$\frac{\sqrt{27}}{3}$	$\frac{3\sqrt{3}}{\sqrt{3}}$	$2\cos 60°$	$\frac{\tan 60°}{2}$	$\frac{\sin 30°}{\cos 30°}$	$3\tan 30°$	$\frac{\sqrt{6}}{\sqrt{2}}$
$\frac{6}{\sqrt{2}}$	$\frac{\cos 60°}{\sin 60°}$	$\frac{9}{3\sqrt{3}}$	$\frac{3}{\sqrt{3}}$	$2\cos 30°$	$\frac{3+\sqrt{3}}{\sqrt{3}}-1$	$3\tan 60°$
$\sqrt{3}$	$\frac{9}{\sqrt{3}}$	$2\sin 60°$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{9}}{\sqrt{3}}$	$\frac{\sqrt{6}}{2}$	$\frac{\cos 30°}{2}$
$3^{\frac{1}{2}}$	$\tan 60°$	$\frac{\sqrt{12}}{2}$	$2\sin 30°$	$\frac{\sin 60°}{\cos 60°}$	$\frac{9^{0.5}}{3^{0.5}}$	$\frac{2\sqrt{6}}{\sqrt{8}}$
$\frac{\cos 60°}{2}$	$\frac{\sqrt{12}}{4}$	$\frac{\sin 30°}{2}$	$\frac{\sqrt{9}}{3}$	$\frac{\tan 60°}{3}$	$\frac{9\times 10^1}{3\times 10^{-1}}$	$\frac{3+\sqrt{3}}{\sqrt{3}}$

Triggy Problems

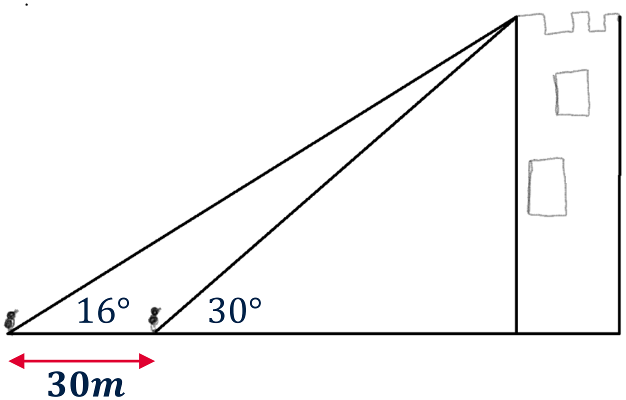
The area of an equilateral triangle is $10\ cm^2$.

What are the lengths of the sides?

Two birds are sitting looking at the top of a tower block, as shown in the diagram

They are 30m apart.

How tall is the tower?



Triggy Problems

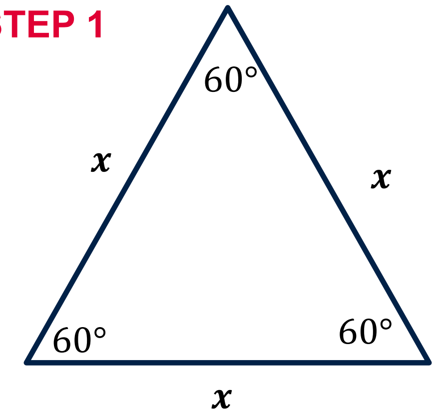
Solutions on the next slide….

Triggy Problems Solutions

The area of an equilateral triangle is $10\ cm^2$.

What are the lengths of the sides?

STEP 1



As this is an equilateral triangle we know all the sides are equal so lets call them x

All the angles are equal so they are all $60°$

STEP 2

We now know 2 sides and an included angle ($60°$)

So we can use the formula $\frac{1}{2}absin\theta = 10$ where $a = b = x$ and $\theta = 60°$

$$\frac{1}{2} \times x \times x \times sin60° = 10$$

$$\frac{1}{2}x^2 \times \frac{\sqrt{3}}{2} = 10$$

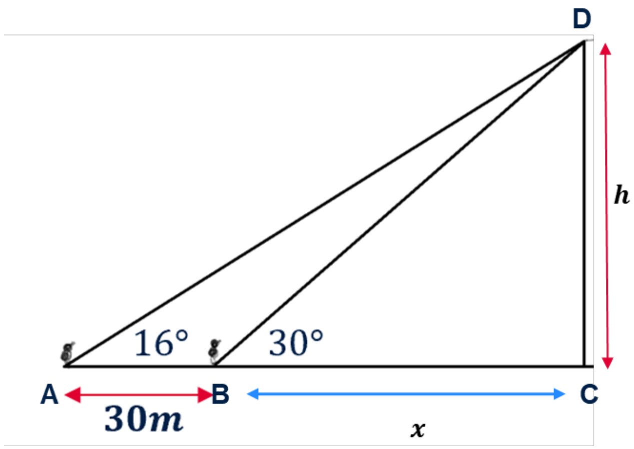
$$x^2 \times \sqrt{3} = 40$$

$$x^2 = \frac{40}{\sqrt{3}}$$

$$\boldsymbol{x = 4.806\ \ to\ 3sf}$$

Triggy Problems Solutions

Two birds are sitting looking at the top of a tower block, as shown in the diagram
They are 30m apart.
How tall is the tower?



Start by labelling the diagram

Height of tower = CD = $\boldsymbol{h}$

Let BC = $\boldsymbol{x}$ so AC = $AB + BC = \ \mathbf{30} + \boldsymbol{x}$

$tan16° = \frac{DC}{AC}$ and $tan30° = \frac{DC}{BC}$

$tan16° = \frac{h}{x+30}$ and $tan30° = \frac{h}{x}$

Rearrange to make h the subject in both expressions

$(\boldsymbol{x} + \mathbf{30})\boldsymbol{tan}\mathbf{16^0} = \boldsymbol{h}$ and $\boldsymbol{xtan}\mathbf{30^0} = \boldsymbol{h}$

As the height is the same we can set these equal to each other

$$(\boldsymbol{x} + \mathbf{30})\boldsymbol{tan}\mathbf{16} = \boldsymbol{xtan}\mathbf{30}$$

$$xtan16 + 30tan16 = xtan30$$

$$30tan16 = xtan30 - xtan16$$

$$30tan16 = x(tan30 - tan16)$$

$$\frac{30tan16}{tan30 - tan16} = x$$

$x = 29.6\ m\ to\ 3sf\ (which\ is\ BC)$

$$Height = xtan30$$
$$Height = 29.6 \times tan30$$
$$Height = 17.1m\ (3sf)$$

Multiple Equations

If $\frac{ab}{a+b} = \frac{1}{4}$ and $\frac{bc}{b+c} = \frac{1}{2}$ and $\frac{ac}{a+c} = \frac{1}{8}$ find a, b and c

Multiple Equations

If $\frac{ab}{a+b} = \frac{1}{4}$ and $\frac{bc}{b+c} = \frac{1}{2}$ and $\frac{ac}{a+c} = \frac{1}{8}$ find a, b and c

Hint:

- Rearrange these equations so they are linear i.e. no fractions
- Find an expression for b and c in terms of a
- Substitute into the equation that uses b and c

Multiple Equations

Follow the link to the solutions

Powers

Using what you know about powers, can you solve this equation

$$(x-6)^{x^2-9}=1$$

Hint available on next slide

Powers

Using what you know about powers, can you solve this equation

$$(x-6)^{x^2-9}=1$$

Hint

- What do you know about a^0
- What do you know about 1^a
- What do you know about $(-1)^a$

Powers

Solutions on the next slide….

Powers Solution

Using what you know about powers, can you solve this equation

$$(x-6)^{x^2-9}=1$$

Case 1: The power is zero

$$x^2-9=0$$

$$x=\pm 3$$

Case 2: The base is 1

$$x-6=1$$

$$x=7$$

Check the power, $7^2 = 49$ and 1 to the power of anything is 1

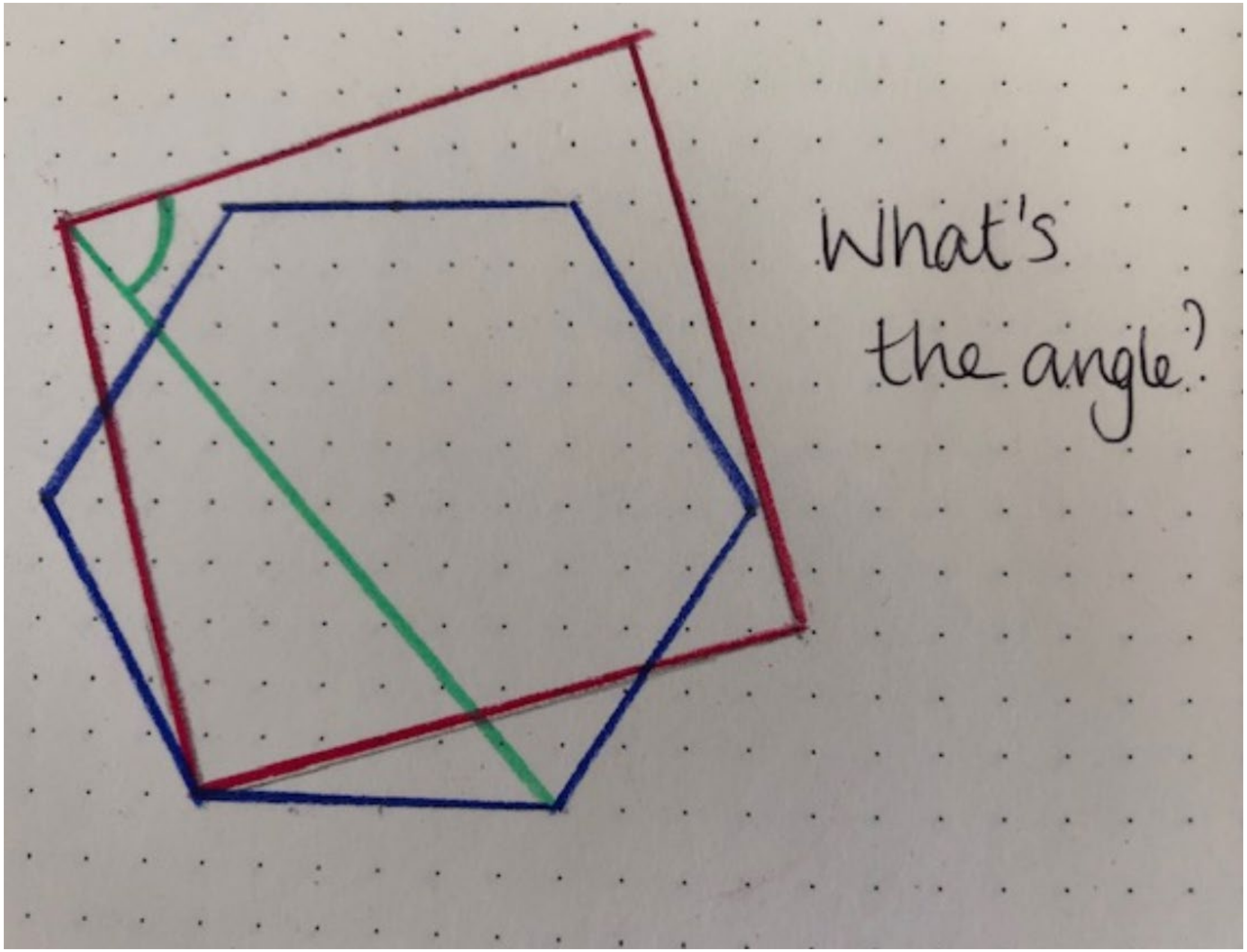
Case 3: The base is -1 (the power must be even)

$$x-6=-1$$

$$x=5$$

Check the power, $x^2-9=25-9=16$

Geometry Puzzle

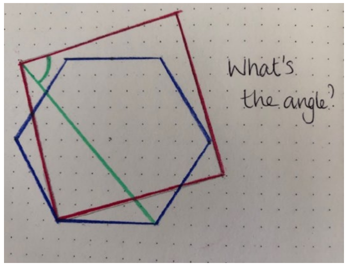


Geometry Puzzle

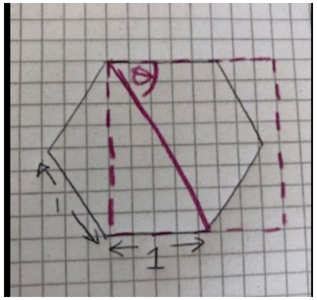
Solutions on the next slide….

Geometry Puzzle Solution

There are many ways to solve this problem - this is just one way!



Rotate the square to the right so that it looks like the second picture

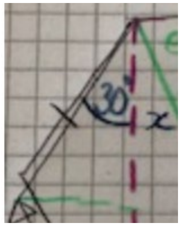


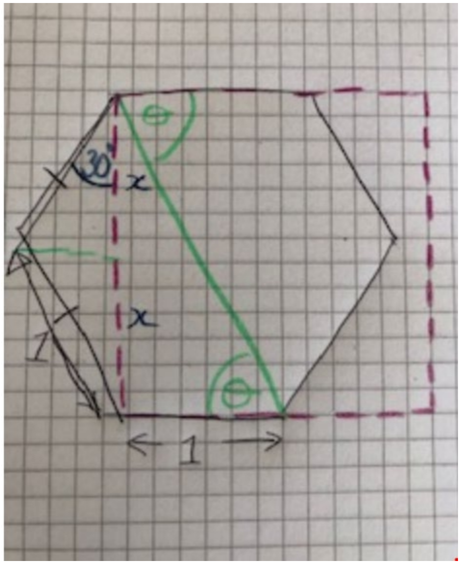
- Let the side length of the hexagon be 1 unit
- An external angle of a regular hexagon is $60°$ - therefore the internal angle is $120°$
- Therefore the base angles of the isosceles triangle inside the hexagon are $30°$

We can now consider this right angled triangle →
to find the length of half of the side of the square $(\boldsymbol{x})$ $cos30° = \frac{x}{1}$

So $\boldsymbol{x} = \frac{\sqrt{3}}{2}$ $\left(as\ \cos 30^0 = x\right)$

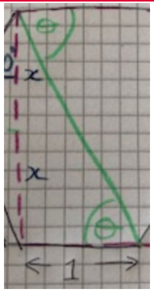
Therefore the side length of the square is $2 \times \frac{\sqrt{3}}{2} = \sqrt{\mathbf{3}}$





Consider the right angled triangle with the height of the square, the base is a side of the hexagon and the hypotenuse is the green line
We can use the fact that alternate angles are equal to get:

$tan\theta = \frac{\sqrt{3}}{1}$ which means $\theta = \tan^{-1}(\sqrt{3})$ so $\boldsymbol{\theta = 60°}$



This puzzle was written by Catriona Shearer if you are interested in more like this look on twitter @Cshearer41

Still want more?

Read about early astronomy and the beginnings of a mathematical science. Essentially it is where trigonometry comes in.

Discover more about 'Trig-om-nom-etry' from the properties of triangles right through to trigonometric function.

Watch this video and learn how equations are used to help us model the environment we live in and make a difference to our lives.

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