

1. $x^4 - 8x^2 + 12 = 0$

$u = x^2$ gives $u^2 - 8u + 12 = 0$
 $(u - 6)(u - 2) = 0$
 $u = 6$ or 2

So $x^2 = 6$ $x^2 = 2$
 $x = \pm\sqrt{6}$ $x = \pm\sqrt{2}$

2. $x - 4\sqrt{x} + 1 = 0$

$u = \sqrt{x}$, $u^2 - 4u + 1 = 0$
 $(u - 2)^2 - 3 = 0$
 $u = 2 \pm \sqrt{3}$

$\sqrt{x} = 2 - \sqrt{3}$ $\sqrt{x} = 2 + \sqrt{3}$
 $\Rightarrow x = (2 - \sqrt{3})^2$ $\Rightarrow x = (2 + \sqrt{3})^2$
 $= 7 - 4\sqrt{3}$ $= 7 + 4\sqrt{3}$

3. $x^6 + 5x^3 = 24$

$u = x^3$, $u^2 + 5u - 24 = 0$
 $(u + 8)(u - 3) = 0$
 $u = -8$ $u = 3$

$\Rightarrow x = -2$ $x = \sqrt[3]{3}$

4. $x^2 - \frac{18}{x^2} = 7$

$x^4 - 18 = 7x^2$

$x^4 - 7x^2 - 18 = 0$
 $u = x^2$; $u^2 - 7u - 18 = 0$
 $(u - 9)(u + 2) = 0$
 $u = 9$ or $u = -2$

$x = 3$ or -3 $x^2 = -2$ (no more solutions)

5. $9^x - 12 \times 3^x + 27 = 0$

$u = 3^x$: $u^2 - 12u + 27 = 0$
 $(u - 3)(u - 9) = 0$
 $u = 3$ or $u = 9$

$3^x = 3 \Rightarrow x = 1$
 $3^x = 9 \Rightarrow x = 2$

$$6. x^8 - 79x^4 = 162$$

$$u = x^4 \quad u^2 - 79u - 162 = 0$$

$$(u - 81)(u + 2) = 0$$

$$u = 81$$

$$u = -2$$

$$x^4 = 81$$

$$x^4 = -2 \quad \times$$

$$x = 3 \text{ or } -3$$

$$7. 3x^{\frac{2}{3}} = 5x^{\frac{1}{3}} + 2$$

$$u = x^{\frac{1}{3}}$$

$$3u^2 - 5u - 2 = 0$$

$$(3u + 1)(u - 2) = 0$$

$$u = -\frac{1}{3}$$

$$u = 2$$

$$x^{\frac{1}{3}} = -\frac{1}{3}$$

$$x^{\frac{1}{3}} = 2$$

$$x = -\frac{1}{27}$$

$$x = 8$$

$$8. 9^{x+1} - 82 \times 3^x + 9 = 0$$

$$\left(\begin{array}{l} u = 3^x ; \quad 9^{x+1} = 9^x \times 9 = 9u^2 \\ \rightarrow 9u^2 - 82u + 9 = 0 \end{array} \right.$$

$$(9u - 1)(u - 9) = 0$$

$$u = \frac{1}{9}$$

$$u = 9$$

$$x = -2$$

$$x = 2$$

$$9. x - \sqrt{x} = 12$$

$$u = \sqrt{x}$$

$$u^2 - u - 12 = 0$$

$$(u - 4)(u + 3) = 0$$

$$u = 4$$

$$u = -3$$

$$\sqrt{x} = 4$$

$$\sqrt{x} = -3 \quad \times$$

$$x = 16$$

$$10. x - 10\sqrt{x+2} + 24 = 0$$

$$(x+2) - 10\sqrt{x+2} + 22 = 0$$

$$u = \sqrt{x+2}$$

$$u^2 - 10u + 22 = 0$$

$$(u - 5)^2 - 3 = 0$$

$$u = 5 \pm \sqrt{3}$$

$$\sqrt{x+2} = 5 - \sqrt{3}$$

$$x+2 = 28 - 10\sqrt{3}$$

$$x = 26 - 10\sqrt{3}$$

$$\sqrt{x+2} = 5 + \sqrt{3}$$

$$x = 26 + 10\sqrt{3}$$