Hidden Quadratics

Watch the video here. Try to reproduce the working for each example.

Questions for you to try.

Remember to check your solutions in the original equation!

1. $x^4 - 8x^2 + 12 = 0$
2. $x - 4\sqrt{x} + 1 = 0$
3. $x^6 + 5x^3 = 24$
4. $x^2 - \frac{18}{x^2} = 7$ [clear fractions first...]
5. $9^x - 12 \times 3^x + 27 = 0$
6. $x^8 - 79x^4 = 162$
7. $3x^{\frac{2}{3}} = 5x^{\frac{1}{3}} + 2$
8. $9^{x+1} - 82 \times 3^x + 9 = 0$
9. $x - \sqrt{x} = 12$
10. $x - 10\sqrt{x+2} + 24 = 0$

Now check your work using the solutions document.

Next up, solving some special quartic equations using a clever trick to make them into hidden quadratics. Watch this video, and then try to repeat the working in the examples. Then do the questions on the next page.

1. Solve:

   a. $x^4 - 4x^3 - 10x^2 - 4x + 1 = 0$

   b. $x^4 - 5x^3 - 10x^2 + 10x + 4 = 0$

   c. $x^4 - 11x^3 + 40x^2 - 66x + 36 = 0$

2. Look at the solutions to your equations in question 1. Why do some of them have four solutions, but others just two? Can you check by plotting a graph (try desmos)? [You can also use desmos to check the numerical values of your answers.

3. This method only works for some quartic equations. We'll see later how to solve quartics more generally (there might be some tough algebra coming, but don't worry if you're finding it hard – one of the purposes of this course is to develop your algebraic fluency) but for now try to think about what kind of quartics we *can* solve with this method. I'm going to point out a couple of features that the equations in question 1 all have in common:

   the constant term is a square number in each case;
   the ratio of the coefficients[1] of x^3 and x is related to this number...

   So this suggests that the form of the quartic has to be

   $$x^4 + bx^3 + cx^2 + abx + a^2 = 0 \qquad (*)$$

   where *a*, *b* and *c* are some constants (in question 1b, $a = -2$, $b = -5$ and $c = -10$).

   a. What are the values of *a*, *b* and *c* in question 1c?

   b. When solving the equation (*) using this method, what will the substitution be to make a quadratic equation?

[1] The *coefficient* of a term is the numerical quantity multiplying the variable. So the coefficient of *x* in the expression $7 - 3x$ is -3.