



KS5 "Full Coverage": Algebraic Fractions (Adding/Subtracting, Partial Fractions and Improper Fractions)

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Simplify single algebraic fractions by factorisation.

[Edexcel C3 June 2006 Q1a] Simplify

$$\frac{3x^2 - x - 2}{x^2 - 1}$$

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Question 2

Categorisation: As above, but where one factor in the denominator is a negation of a factor in the numerator.

[OCR C4 June 2012 Q1i] Simplify

$$\frac{1 - x}{x^2 - 3x + 2}$$

.....

Question 3

Categorisation: Add/subtract fractions where prior factorisation of a denominator is required.

[Edexcel C3 June 2017 Q1] Express

$$\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$$

as a single fraction in its simplest form.

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Question 4

Categorisation: As above, but with more than two fractions.

[Edexcel C3 June 2014(R) Q1]

Express

$$\frac{3}{2x + 3} - \frac{1}{2x - 3} + \frac{6}{4x^2 - 9}$$

as a single fraction in its simplest form.

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Question 5

Categorisation: As above, but with a non-fractional term.

[Edexcel C3 Jan 2007 Q2a Edited]

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2$$

Show that $f(x) = \frac{x^2+ax+b}{(x+2)^2}$, $x \neq -2$ where a and b are constants to be found.

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Question 6

Categorisation: As above.

[Edexcel C3 June 2009 Q7a Edited]

The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$$

$x \in \mathbb{R}$, $x \neq -4$, $x \neq 2$

Show that $f(x) = \frac{A}{x-2}$ where A is an expression to be found in its simplest form.

$A = \dots\dots\dots$

Question 7

Categorisation: Factorise expressions using long division.

[Edexcel C3 Jan 2007 Q7c]

$$f(x) = x^4 - 4x - 8$$

Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants a , b and c .

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Question 8

Categorisation: Use long-division to split a top-heavy fraction into a quotient and remainder.

[Edexcel C3 Jan 2008 Q1]

Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)}$$

find the values of the constants a , b , c , d and e .

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Question 9

Categorisation: As above.

[Edexcel C3 June 2016 Q6a]

$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, x > 2, x \in \mathbb{R}$$

Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B .

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Question 10

Categorisation: Further practice.

[Edexcel C3 June 2013 Q1]

Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4},$$

where $x \neq \pm 2$, find the values of the constants a, b, c, d and e .

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Question 11

Categorisation: Split a fraction into partial fractions.

[Edexcel A2 SAM P2 Q16a] Express

$$\frac{1}{P(11 - 2P)}$$

in partial fractions.

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Question 12

Categorisation: As above, but with a repeated factor (where the form required is given).

[Edexcel C4 June 2012 Q1a]

$$f(x) = \frac{1}{x(3x - 1)^2} = \frac{A}{x} + \frac{B}{3x - 1} + \frac{C}{(3x - 1)^2}$$

Find the values of the constants A , B and C .

.....

Question 13

Categorisation: As above, but where guidance on the required form is not given.

[Edexcel C4 June 2014(R) Q4a] Express

$$\frac{25}{x^2(2x + 1)}$$

in partial fractions.

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Question 14

Categorisation: Further practice of the above.

[OCR C4 June 2012 Q9i]

Express

$$\frac{x^2 - x - 11}{(x + 1)(x - 2)^2}$$

in partial fractions.

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Question 15

Categorisation: Partial fractions involving a top-heavy fraction.

[Edexcel C4 June 2010 Q5a]

$$\frac{2x^2 + 5x - 10}{(x - 1)(x + 2)} \equiv A + \frac{B}{x - 1} + \frac{C}{x + 2}$$

Find the values of the constants A , B and C .

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Question 16

Categorisation: As above.

[Edexcel C4 Jan 2013 Q3] Express

$$\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$$

in partial fractions.

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Question 17

Categorisation: Bonus question!

[OCR C4 June 2015 Q10ii Edited]

It can be shown that

$$\frac{x + 8}{x(x + 2)} \equiv \frac{4}{x} - \frac{3}{x + 2}$$

By first using division, express

$$\frac{7x^2 + 16x + 16}{x(x + 2)}$$

in the form

$$P + \frac{Q}{x} + \frac{R}{x + 2}$$

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Answers

Question 1

$$\frac{3x+2}{x+1}$$

Question 2

$$-\frac{1}{x-2}$$

Question 3

$$\frac{2}{x-3}$$

Question 4

$$\frac{2}{2x+3}$$

Question 5

$$a = 1, b = 1$$

Question 6

$$A = x - 3$$

Question 7

$$a = 2, b = 4, c = 4$$

Question 8

$$a = 2, b = 0, c = -1, d = 1, e = 0$$

Question 9

$$A = 3, B = 4$$

Question 10

$$a = 3, b = -2, c = 7, d = -8, e = 24$$

Question 11

$$\frac{1}{11P} + \frac{2}{11(11-2P)}$$

Question 12

$$A = 1, B = -3, C = 3$$

Question 13

$$-\frac{50}{x} + \frac{25}{x^2} + \frac{100}{2x+1}$$

Question 14

$$-\frac{1}{x+1} + \frac{2}{x-2} - \frac{3}{(x-2)^2}$$

Question 15

$$A = 2, B = -1, C = 4$$

Question 16

$$3 + \frac{2}{x+2} - \frac{1}{3x-1}$$

Question 17

$$P = 7, Q = 8, R = -6$$